Two proofs that
$(A \Rightarrow B)$ if and only if $(\neg B \Rightarrow \neg A)$

Direct proof

We can construct a "truth table" that maps the situation for all possible values of **A** and **B**:

A	**B**	$A \Rightarrow B$	$\neg B$	$\neg A$	$\neg B \Rightarrow \neg A$
T	T	**T**	F	F	**T**
T	F	**F**	T	F	**F**
F	T	**T**	F	T	**T**
F	F	**T**	T	T	**T**

The values in the column of $A \Rightarrow B$ and $\neg B \Rightarrow \neg A$ are the same, so they are equivalent: that is an ***if and only if***.

Proof by contradiction *(reductio ad absurdum)*

We suppose that the hypothesis is true but the thesis is false, we reach a contradiction.

We first suppose that

(A ⇒ B) is true but **(¬B ⇒ ¬A)** is not true

If this were so, then we could have

(A ⇒ B) true

¬**B** true and ¬**A** false (because **(¬B ⇒ ¬A)** is not true)

That is, we could have

(A ⇒ B) true

¬**B** true

A true (because we supposed ¬**A** false)

But then

A true and **(A ⇒ B)** true imply that **B** is true

while we also supposed that ¬**B** true

This leads to a contradiction: both **B** and ⌐**B** should be true.

Therefore we must have that

If (A ⇒ B) then (⌐B ⇒ ⌐A)

The vice versa, that is

If **(⌐B ⇒ ⌐A)** then **(A ⇒ B)** is proven in the same way: just

substitute **A** for ⌐**B** and

substitute **B** for ⌐**A**.

After all, **A**'s and **B**'s are just placeholders - meaningless marks on paper...