Mathematics

The BIG game
Behind the little tricks

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(She/Her)

Hi there! :-)

The goal of this talk is to show maths is nothing to fear, but it's a tool to embrace to empower yourself (and have fun too). Don't worry about what society told you: just relax and be ready for a journey (or a travel guide, more like – this is just a talk).

Our teacher began by saying that
French has very difficult sounds
to pronounce, for instance *en*.
She repeated this sound three times,
then asked each student to replicate it.
Then she declared that **we would have**
never really learned French.

(Peter Bichsel)

“School” mathematics is often taught in a punitive way, as a succession of exercises designed to trick you into failing. This is usually because exercises of that kind are perfect for giving grades that pigeonhole people.

But mathematics is a creative endeavour, not much different from a language. How would you feel if someone taught you only the difficulties of the grammar and never mentioned the possibility of a literature? Wouldn't you hate that language?

What is mathematics?

$$15^4\ x + 143x^2 + 43 + 7x = 42$$

NO.

(Not good mathematics, at least.)

OK, technically this **is** mathematics.

But it's mostly something made to trick you into failing an exam.

A good part of mathematics was invented to avoid calculations: abstract from the single case, find a general rule, solve the problem once and for all.

What is mathematics?

$$(A \Rightarrow B)$$
$$\Leftrightarrow$$
$$(\neg B \Rightarrow \neg A)$$

That's more like it.

OK, this is a cheap theatrical trick on my part. Sorry.

You can imagine this as being presented with a very long word in a foreign language - but a word that once you grasp the very basics of that language makes sense.

Please forgive the cheap trick and bear with me. You will not just understand this slide, you will OWN what it says – in your mind. (With a little help from the handouts, if needed. This is not a test.)

A Hitchhiker's Guide To The Galaxy

DON'T PANIC

is always in line with a geekfest.

Keep calm and do maths.

(Sorry.)

*Mathematics is a **game** played according to **simple rules** with **meaningless marks on paper***

(David Hilbert)

This is possibly my favourite quote about mathematics.

You can learn to do mathematics just as you learn to play a board game. Or to speak Klingon, or Quenya.

The approach that works for you is the right one. You can combine the two, of course.

See? It's as if maths wants you to learn it!

Is it worth it?

The romantic argument!

You learn a **new language**
that gives you **new ideas**.

You **discover a world**
of **beauty** in your mind.

Yes, I said *beauty*.

“But Maths is objective, beauty is subjective!”

Well...

Two mathematicians can disagree on the beauty of a theorem, or a formula. (They do. A lot.)

Here's a debate on Quora about “which is the most beautiful mathematical equation” - but really, just ask Google and you'll hit a dozen mathematicians arguing.

www.quora.com/What-is-the-most-beautiful-equation

Three examples.

$$e^{i\pi} + 1 = 0$$

$$i^2 = j^2 = k^2 = -1$$

$$\frac{\partial \mathcal{L}}{\partial f} - \Sigma_i \frac{\partial}{\partial x_i} \frac{\partial \mathcal{L}}{\partial f_{x_i}} = 0$$

I don't have time to explain you these. BUT! Look at how “general” they are (almost no numbers), how “plain” they look (especially the first two: there aren't many calculations in there)…

And they are quite different. As I said, it's a matter of taste.

The first one (Euler's Identity) is usually the winner in the “most beautiful” polls.

For an extensive collection of biographies, go to www-history.mcs.st-and.ac.uk - or Wikipedia!

Yes, there are women (Sofia Kovalevskaya; Sophie Germain; Emmy Noether); two people with disability (Niccolò Fontana Tartaglia – “Tartaglia” is a nickname that means “stutter”; Leonhard Euler, of “most beautiful equation” fame, went blind in his later years); there are people who are not white (Muhammad ibn Musa al-Khwarizmi, hence the word “algorithm”; the Persian poet Omar Khayyam) – although maths dates back to when “white European” people were the “uncivilized” ones...

Remember: mathematics is many different things.

NOT THIS.

Maths is not Soylent.

Maths is a banquet.

...so it's easier to find something you like!

Important: it doesn't necessarily get harder. Actually, the first topics are the hardest, because you have to get used to the whole new thing. Also, “easy” and “hard” are very subjective – just like “beautiful”. It happens in your mind, and your mind is unique.

Also: even if there are topics that are needed to understand something, there isn't a path that you have to follow strictly. Sometimes you can reach the same topic from two different sides, and one clarifies the other. In other words: a world to explore, not a series of checkpoints.

Is it worth it?

The moral/ethical argument!

You learn to be honest

and challenge your assumptions.

Mathematics is based on not giving anything for granted: everything you do must be explicitly stated in the hypothesis of the theorem, in the definitions, or in the axioms.

A nice (although technical) example of an application to politics and sociology: a data-based study that measures the sexist bias in everyday language, or more precisely in our mental associations.
https://www.technologyreview.com/s/602025/how-vector-space-mathematics-reveals-the-hidden-sexism-in-language/#

Here's David Hilbert again.

Mathematics knows no races or boundaries. For mathematics, ***the world is one country****.*

It's powerful.

It can be shared.

It must be shared.

Especially if you've been denied power over yourself.

And yes, I do love David Hilbert.

Six habits of highly mathematical people.

Discussing **definitions**.

Evaluating **many possible consequences** of a claim.

Tearing apart the assumptions underlying an argument.

Coming up with **counterexamples**.

Being wrong often and admitting it.

Scaling the ladder of **abstraction**.

(by Jeremy Kun)

From: medium.com/@jeremyjkun/habits-of-highly-mathematical-people-b719df12d15e#.6y4g8qpiy

It's very interesting and very true, at least in my experience (and the experience of the mathematics professor who linked it on Twitter).

So... **LET'S PLAY!**

$$(A \Rightarrow B)$$
$$\Leftrightarrow$$
$$(\neg B \Rightarrow \neg A)$$

I told you you'd end up owning this thing in your mind...

Again…

DON'T PANIC

> *In mathematics,* ***you don't understand things. You get used to them.***
>
> (John von Neumann)

My other favourite quote about mathematics, from possibly the greatest mathematician of the 20th century. His biography on Wikipedia gives you an idea of how much “mathematics” is an umbrella term.

I think it's one of the most dangerous misunderstanding that scares people away from mathematics: you can reach the point where you *perceive* mathematical ideas as you perceive the world – but don't expect them to make sense.

By the way, a priest told me that it happens the same with theology.

Von Neumann had an interesting life, who literally shaped the 20th Century – not always for the best. You can read about him on Wikipedia.

Goal of the game.

Something can be **true** or **false**.

But **you don't know** if it is.

Aren't you even a bit **curious?**

So, let's play. We're going to see the rules.

As I said before: don't worry about *understanding*. It's not like you *understand* the rules of poker, you just play it. (If you know the rules, or you can look them up easily, of course.)

The game.

You **assume a few things to be true, and declare them**. **(Axioms)**

You make up **precise ideas** and give them **names**. **(Definitions)**

You **connect the ideas**. **(Theorems)**

How it works!

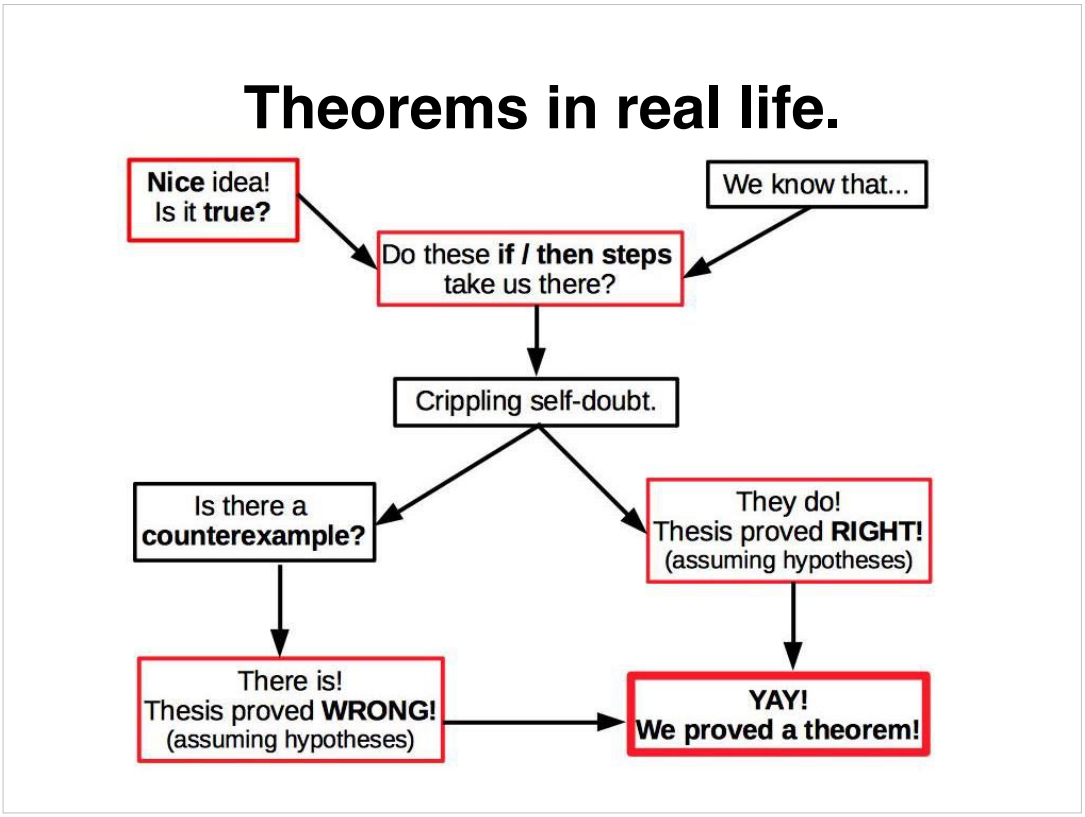
Theorems in theory.

You declare the **boundaries** of where your theorem will hold. **(Hypothesis)**

You state your **thesis**. **(Thesis)**

You show the connection. **(Proof)**

In theory...



And in practice.

This is a wonderful article that tells you a bit more about how mathematicians work. Spoiler: Hollywood usually gets it wrong. (*Big Hero 6* gets it pretty well, though.)

www.forbes.com/sites/quora/2016/07/29/the-poetry-and-process-behind-every-mathematical-proof/#169b74551ae7

Today's game: logic!

Our basic pawns are called **statements.**

They can have value True or False.

We denote them with capital letters like **A**, **B**, **C...**

A $\wedge$ **B** is the statement “both **A** and **B** are true” ***(and)***

A $\vee$ **B** is the statement “**A** and/or **B** are true” ***(or)***

$\neg$ **A** is the statement **“A** is false” ***(not)***

A few pawns and a few more rules for today's game!

Two more definitions

A ⇒ B is the statement

"**if A** happens, **then B** must happen" **(*if / then*)**

A ⇔ B is the statement

"**if A** happens, **then B** must happen **and vice versa**"

(*if and only if*)

Another two meaningless symbols…

Two axioms

Axiom: the statement **(A** $\wedge$ $\neg$**A)** is false.

(**A** cannot be both true and false.)

Axiom: the statement **(A** $\vee$ $\neg$**A)** is true.

(A must be either true or false**)**

Two things we give for granted. They might sound intuitive (they are, in my experience of the world), but we have to state them anyway.

A theorem: “contraposition”

(A ⇒ B)

if and only if

(¬B ⇒ ¬A)

FIND **YOUR** WAY!

Your theorem!

There are two possible proofs in the handouts, but many more are possible. You can find another one, maybe.

Yes, I love David Hilbert quotes.

We must know.
We will know!

(SPOILER: Sorry, David.)

Do you feel powerful?

Historical note: the quote “We must know, we will know” is written on Hilbert's grave too.

Problem… it's not that simple. Never take things for granted...

Gödel's incompleteness theorem

If a formal system is **consistent** and complex
enough to **contain arithmetic**,
then it is **incomplete**; that is,
there are statements that
cannot be proven to be true or false.

...not even that you can prove things.

“Formal system” basically means “a bunch of axioms and definitions”. “Consistent” means that there are no contradiction. “Complex enough to contain arithmetic” means that you can do 1+1.

Then there are statements that are true or false (because they're statements, it's their nature), but there is no proof of whether they are true or false.

Is it bad?

Now you know new things
including that you won't know everything
but you don't know what you won't know.

Don't you want to know
how far you can go?

Isn't that exciting?

Yes, maybe a bit scary. But don't you want to take up the challenge?

Thank you!

For comments / questions / suggestions: you can reach me on Twitter @mmcasetti and my website is www.planningadinner.net